Entanglement at quantum criticality in an Anderson model with a pseudogap

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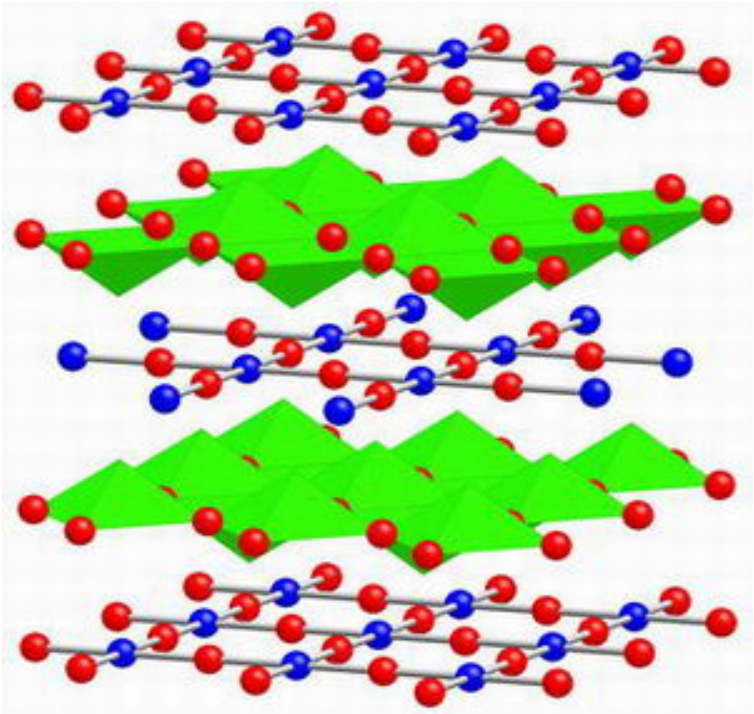
Strongly-Correlated Electron Systems

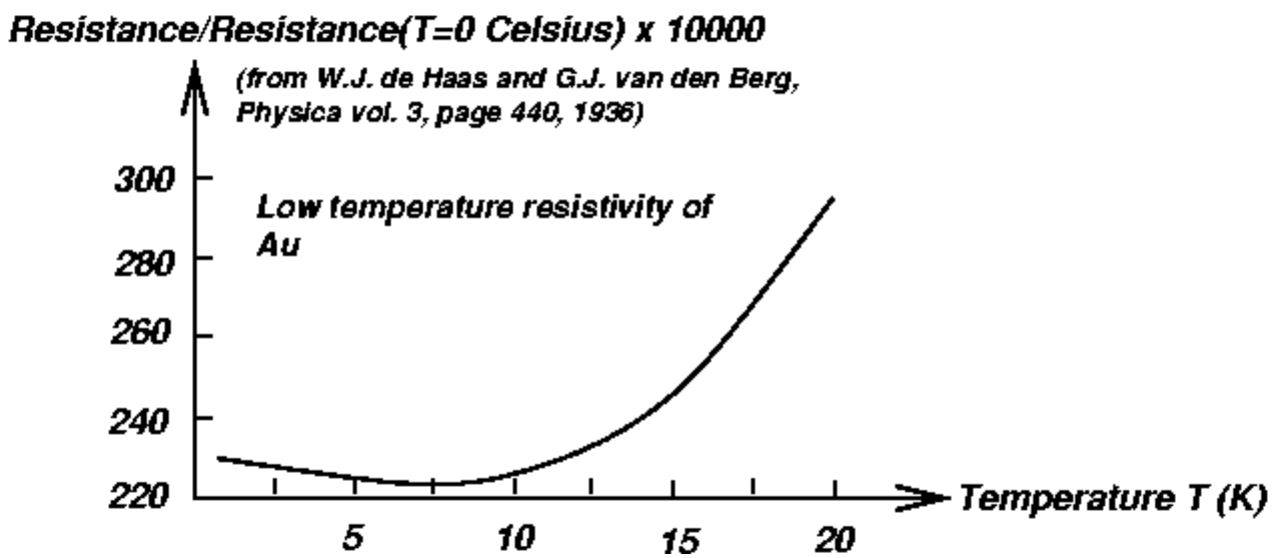
- Negligible electron interaction → Fermi-liquid
- Strong electron interaction → Strongly-correlated
  - High-T_c superconductivity
  - Colossal magnetoresistance
  - Kondo effect



Argonne National Laboratory

Quantum-Impurity Systems

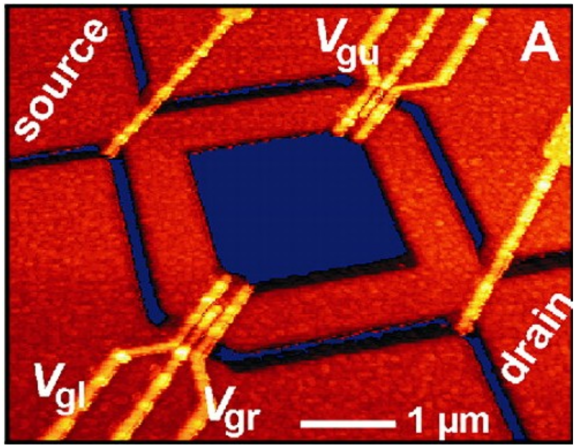
- Pioneered to study the Kondo effect – anomolous increase in resistance at low-temperatures in metals



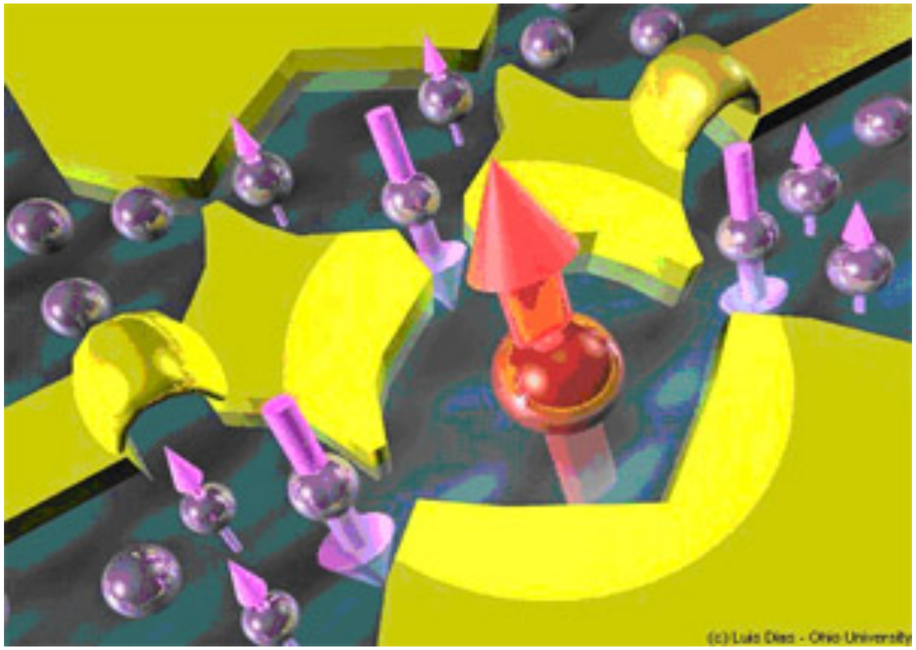
- Local degree of freedom - a magnetic impurity
- Bulk degrees of freedom (macroscopic host) – conduction band in a metal

Quantum Dots

- Engineered semiconductor nanostructures
- "Artificial atoms"



Van der Wiel et al., Science (2000)



Luis Dias, Ohio University

- Magnetic impurity → Quantum dot
- Bulk metal → Semiconductor 2DEG

Entanglement and Quantum Information

- Entanglement

$$|\Psi\rangle \neq |\psi_1\rangle_1 \, |\psi_2\rangle_2$$

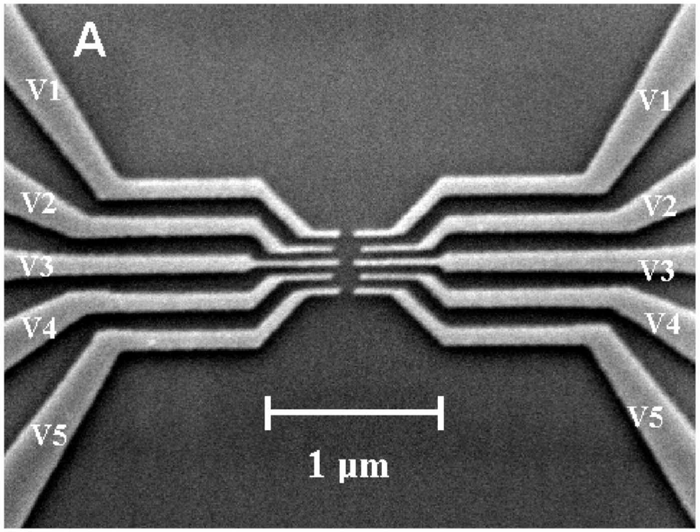
- Example:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} |\uparrow\rangle_1 \, |\downarrow\rangle_2 - \frac{1}{\sqrt{2}} |\downarrow\rangle_1 \, |\uparrow\rangle_2$$

- Entangled qubits

  → Quantum computer

- Quantum dots?



Jeong et al., Science (2001)

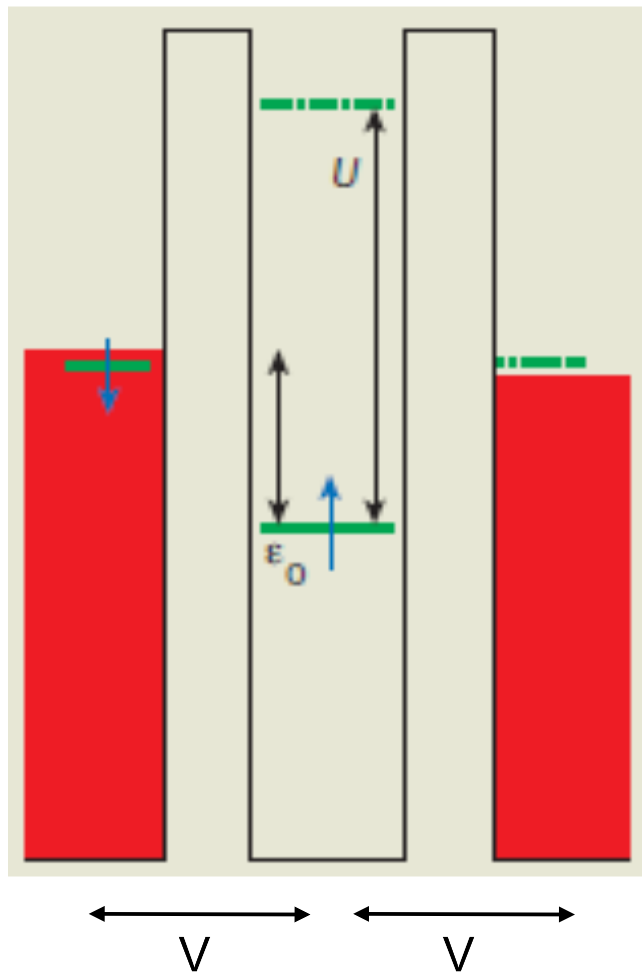
The Pseudogap Anderson Model

- Impurity level
  - Singly occupied – energy $\varepsilon_d<0$
  - Doubly occupied – energy $2\varepsilon_d+U$
- Pseudogap Density of States

$$\rho(\epsilon) = \begin{cases} \rho_0 |\frac{\epsilon}{D}|^r, & |\epsilon| \leq D \\ 0, & |\epsilon| > D \end{cases}$$

- Tunneling Coefficient V

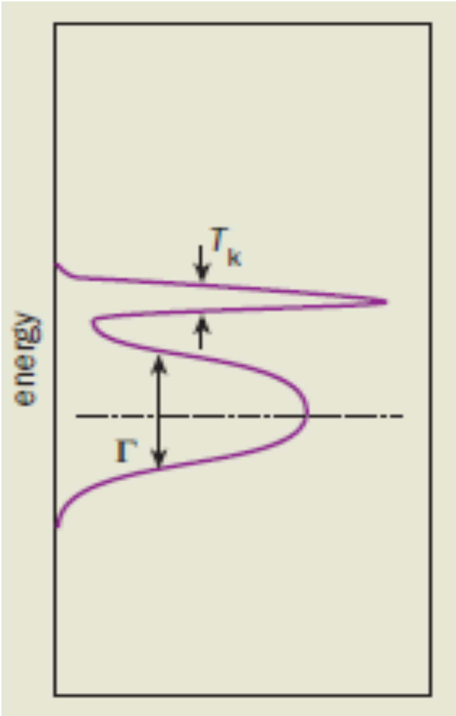
Kouwenhoven et al., Physics World (2001)



Quantum Phase Transitions

- Thermal fluctuations → classical phase transition
- Quantum mechanical fluctuations (from the uncertainty principle) → quantum phase transition (QPT)
- QCP – local-moment phase vs strong-coupling phase (Kondo effect)
- We vary the hybridization width:

$$\Gamma = \pi \rho_0 V^2$$



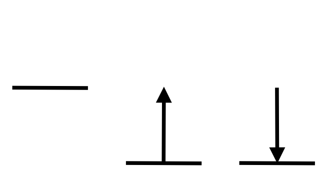
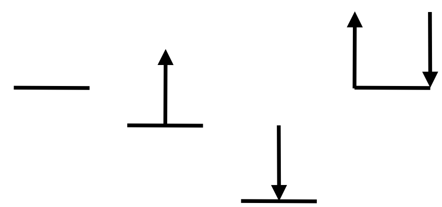
Kouwenhoven et al., Physics World (2001)

Entanglement Entropy at QCP

- Definition:

$$S_e = -\sum_i p_i \ln p_i$$

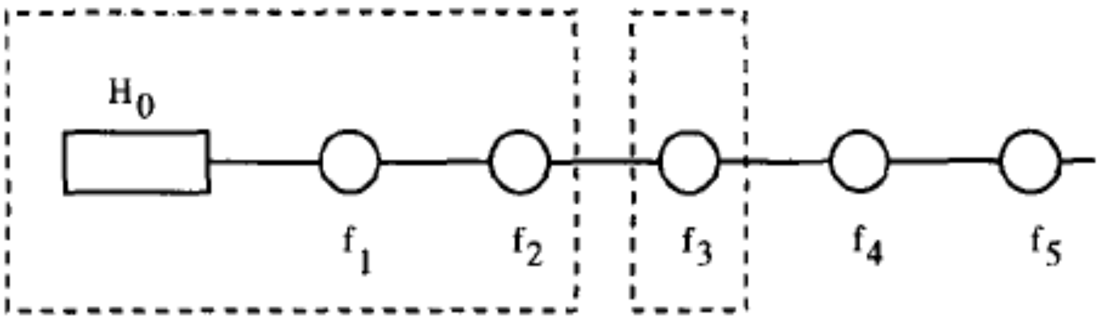
- Measure of information and entanglement
- Degeneracies → S_e artificially large

  Small perturbation takes  to 

- Study entanglement entropy at QCP
- Computational Method: Numerical Renormalization Group

NRG Method

- Main idea: transform the Hamiltonian into a "chain"

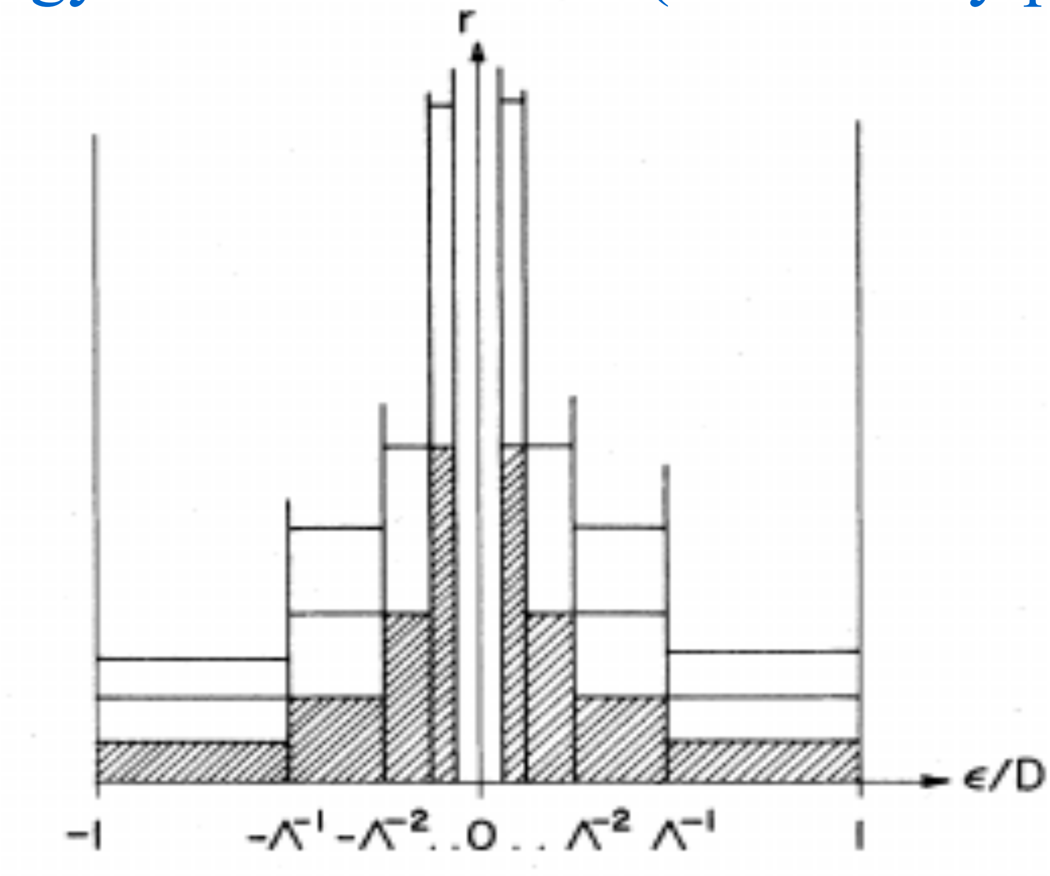


Costi, Wilson's NRG (1999)

- Each element of the chain couples only to its nearest neighbors
- As Hamiltonian gets too large, begin discarding high energy terms

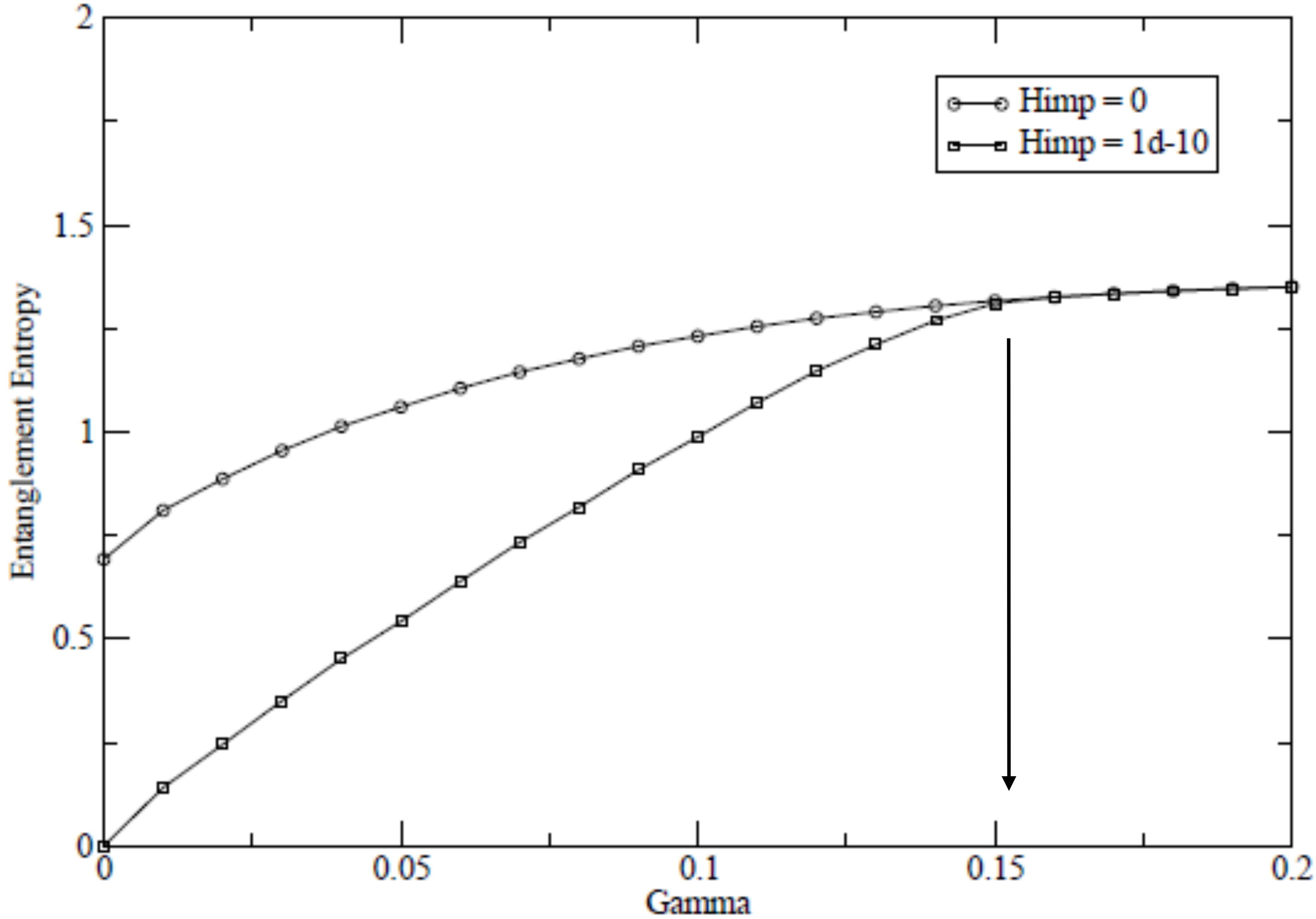
NRG Method (Cont.)

- Discretize k-space using Wannier states
- Higher energy = more localized (uncertainty principle)

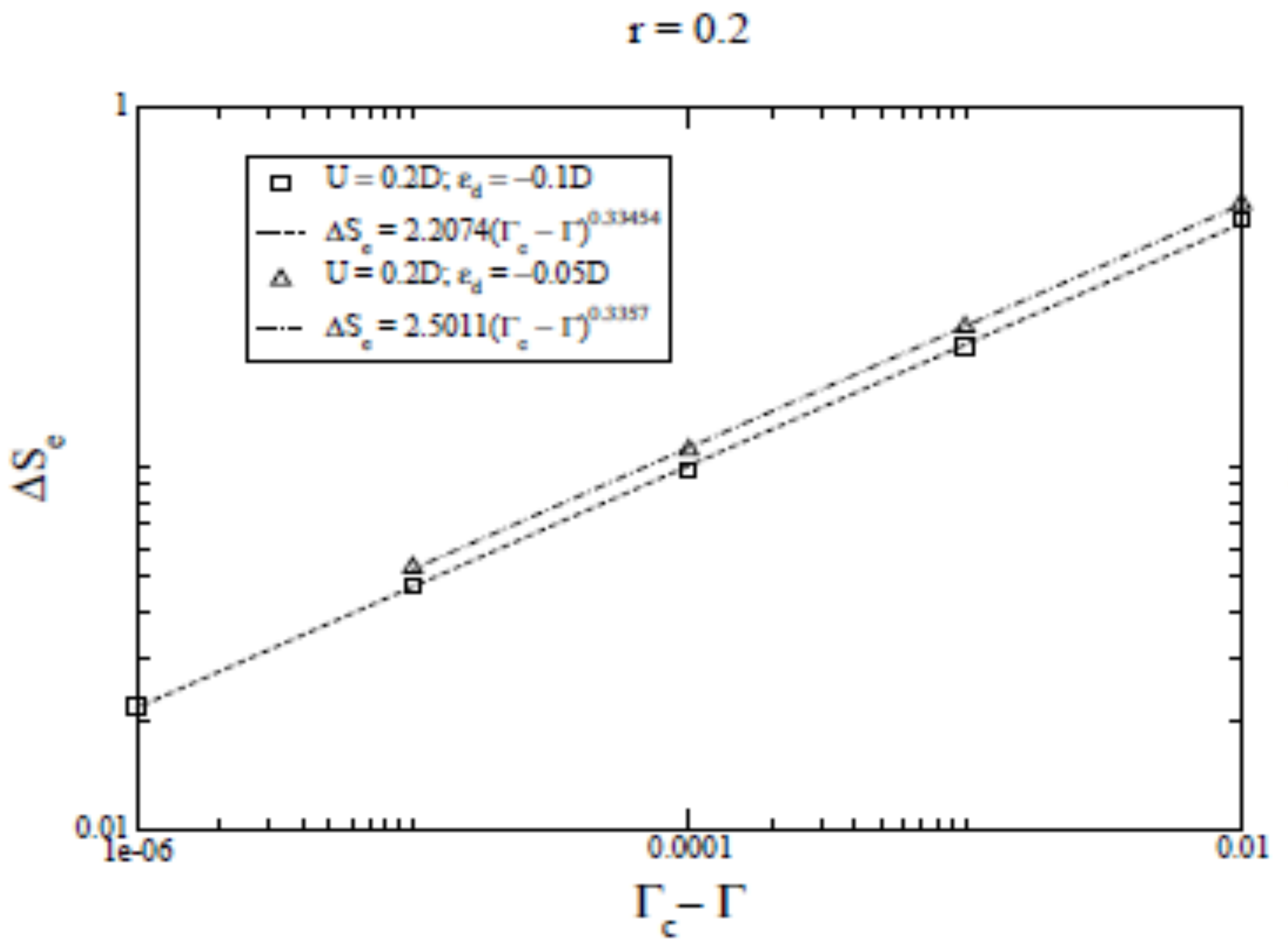


Krishna-murthy et al., Phys. Rev. B. (1980)

Results

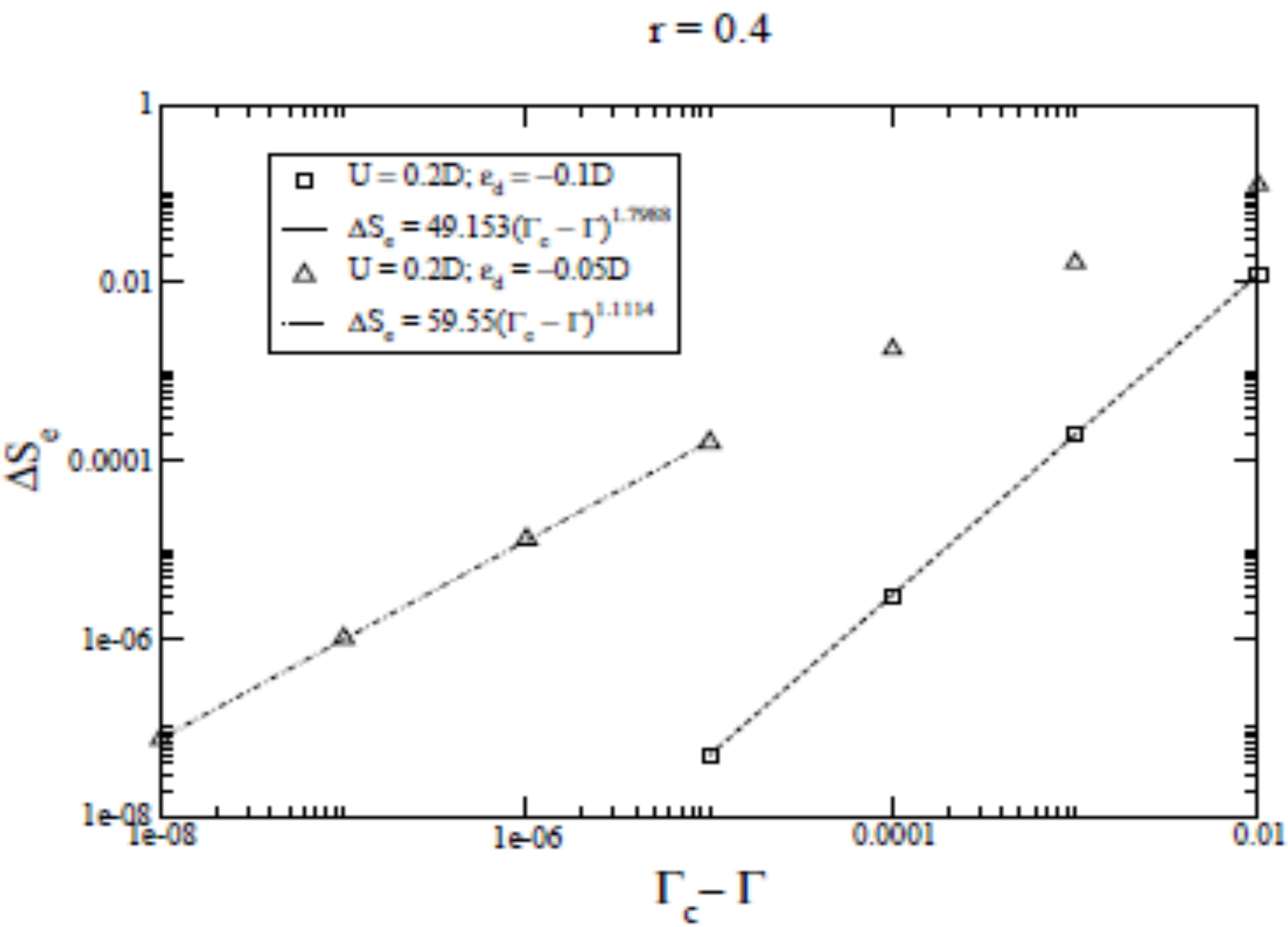

Himp = 0
Himp = 1d-10
Entanglement Entropy
Gamma
2
1.5
1
0.5
0
0
0.05
0.1
0.15
0.2

Results



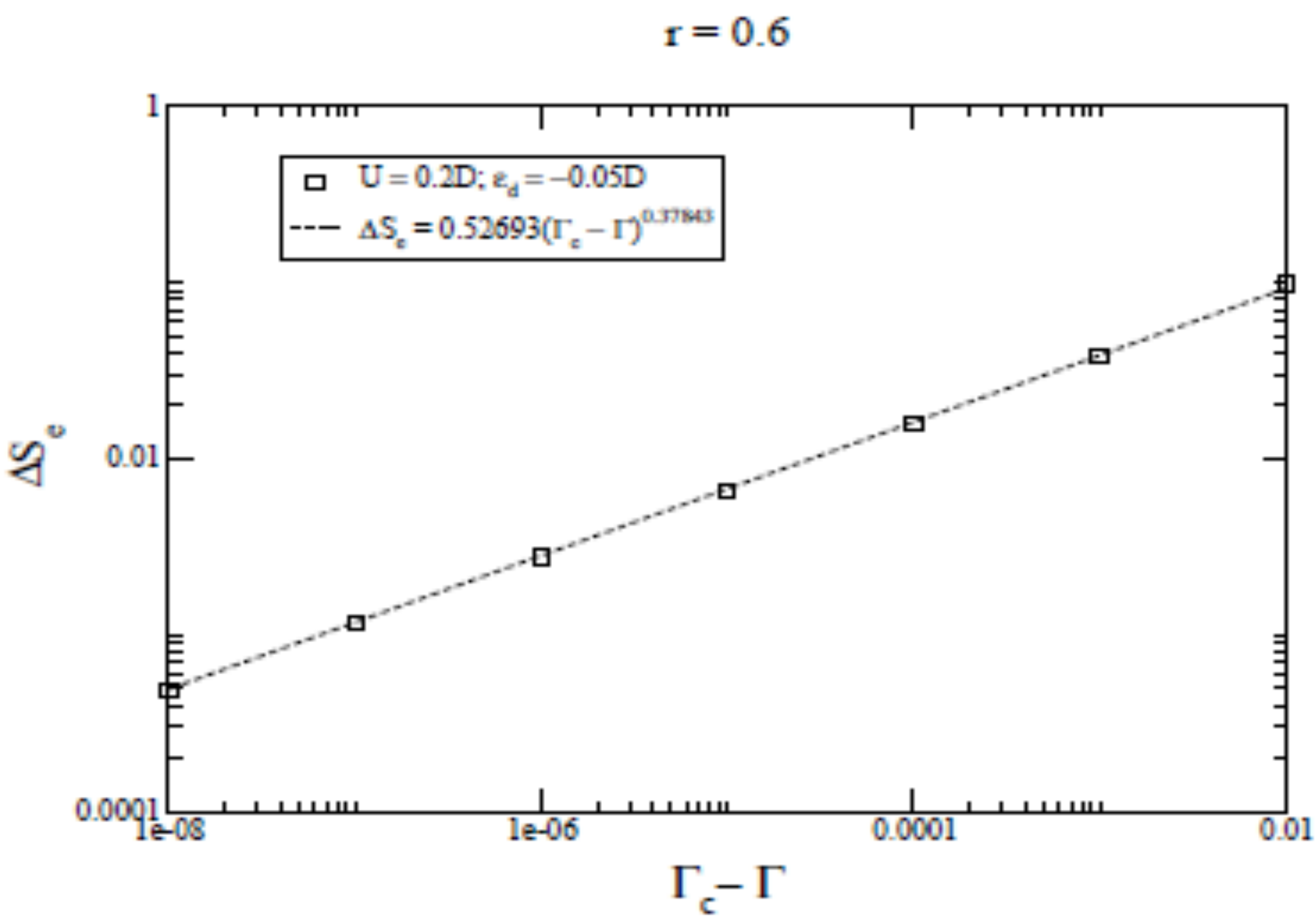
r = 0.2
U = 0.2D; ε_d = −0.1D
ΔS_e = 2.2074(Γ_c − Γ)^0.33454
U = 0.2D; ε_d = −0.05D
ΔS_e = 2.5011(Γ_c − Γ)^0.3357
ΔS_e
1
0.01
1e-06
0.0001
0.01
Γ_c − Γ

Results



r = 0.4
U = 0.2D; ε_d = −0.1D
ΔS_e = 49.153(Γ_c − Γ)^1.7988
U = 0.2D; ε_d = −0.05D
ΔS_e = 59.55(Γ_c − Γ)^1.1114
ΔS_e
1
0.01
0.0001
1e-06
1e-08
1e-08
1e-06
0.0001
0.01
Γ_c − Γ

Results



r = 0.6
U = 0.2D; ε_d = −0.05D
ΔS_e = 0.52693(Γ_c − Γ)^0.37843
ΔS_e
Γ_c − Γ
1
0.01
0.0001
1e-08
1e-06
0.0001
0.01

Results

$$S_e(h=0) - S_e(h\to 0) \propto (\Gamma_c - \Gamma)^{\eta}$$

r	ϵ_d	η	2β
0.2	−0.1	0.3345	0.3200
0.2	−0.05	0.3357	0.3200
0.4	−0.1	1.7988	1.8280
0.4	−0.05	1.0957	1.16
0.6	−0.05	0.37843	0.376

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- University of Florida

References

[1] A. J. Schofield, Contemporary Physics , 95 (1999).

[2] L. Kouwenhoven and L. Glazman, Physics World , 33 (2001).

[3] C. Gonzalez-Buxton and K. Ingersent, Phys. Rev. B **57**, 14254 (1998).

[4] K. Le Hur, P. Doucet-Beaupre, and W. Hofstetter, Phys. Rev. Lett. **99**, 126801 (2007).

[5] N. Mathur, F. Grosche, S. Julian, I. Walker, D. Freye, R. Haselwimmer, and G. Lonzarich, Nature **394**, 39 (1998).

[6] D. M. Broun, Nature Physics **4**, 170 (2008).

[7] T. A. Costi, "Wilson's numerical renormalization group," in *Density-Matrix Renormalization - A New Numerical Method in Physics*, Lecture Notes in Physics, Vol. 528, edited by I. Peschel, X. Wang, M. Kaulke, and K. Hallberg (Springer-Verlag, 1999) Chap. 1.

[8] K. Ingersent and Q. Si, Phys. Rev. Lett. **89**, 076403 (2002).